is explained by the fact that very steep temperature gradients exist at the electrode surfaces, raising the temperature across any ion sheath to a value approaching that of the bulk gas. The last point follows also from the fact that the ion density calculated from the random ion currents approaches the electron density deduced from the bulk gas conductivity.

It should be expected from these results that saturation effects in MHD experiments should be primarily controlled by the random ion current unless special electrode materials with low work functions are used.

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Hypersonic Slip Flow past the Leading Edge of a Flat Plate

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Nomenclature

c_1 = constant, $c_1 = (\pi/2)^{1/2}$
M = Mach number
T = temperature
v = velocity
x, y = system of coordinates
ξ = $\xi = v_n/v_{n\infty}$
η = $\eta = (1 - \xi)/(1 - \xi_0)$
ξ_0 = ξ value corresponding to the state given by Rankine-Hugoniot relations
φ_b = dimensionless slip velocity
λ = mean free path

Subscripts

∞ = refers to freestream condition
a = refers to the state between the compressive and boundary-layer region
b = refers to the slip condition
n = refers to the direction normal to the wave direction

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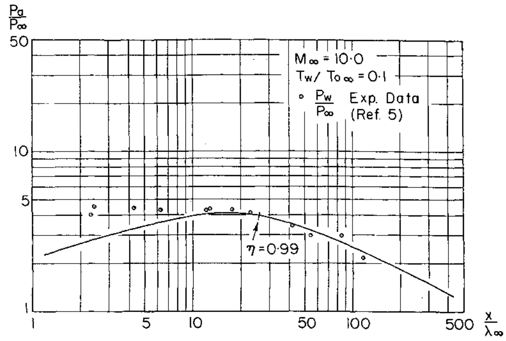

Fig. 1 The pressure distribution along the flat plate immersed in a hypersonic rarefied flow.

REPORTED here are some of the results obtained from the study on hypersonic low-density flow past the leading edge of a flat plate. The method of analysis of this study is based on an integral approach.

It has been recognized that,[1,2] close to the leading edge of the flat plate, the shock layer and the boundary layer are merged and indistinguishable, and this is termed as the merged layer region. It also has been well established that shock wave is not thin when it occurs in rarefied flows. Therefore, close to the leading edge of the flat plate, one may divide the disturbed flowfield into two subregions, namely, the continuous viscous-conducting compressive region (shock-wave-like region), and the shock-viscous-layer region (boundary-layer-like region). It was assumed that these two regions are joined together under the criterion that they shall have the same velocity (horizontal component) along their common boundary.

For the continuous compressive region, a wave direction (similar to the wave angle of a shock wave) may still be defined which is normal to the direction along which the compression takes place, and a new parameter termed as "degree of compression" may be defined to signify the extent of compression since this compression process may not necessarily be terminated to the states given by Rankine-Hugoniot relations. To facilitate the calculations, it was further assumed that the shock-wave-like region is locally straight (i.e., the velocity component along the wave direction is constant across this region at the section considered, and the effect of wave curvature is ignored), and the Navier-Stokes equation is applicable to this region so that Becker's analysis[3] may readily be adopted. Furthermore, the Prandtl number of the fluid was assumed to be $\frac{3}{4}$ so that the velocity-temperature relationship for the fluid within this region can be expressed in closed form.

For the shock-viscous layer region, a linear slip-velocity profile was employed. Since the Crocco integral relationship has been shown[1,2] to provide good approximations for this flow region, it also has been adopted here to correlate the velocity and temperature profiles within this region which eliminates the necessity of the consideration of the energy relationship, and therefore greatly simplifies the analysis.

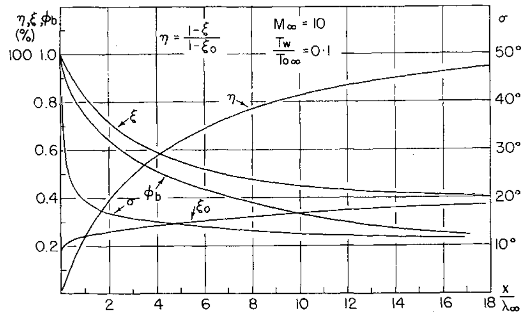

Fig. 2 The wave angle, the degree of compression, and slip velocity along the flat plate.

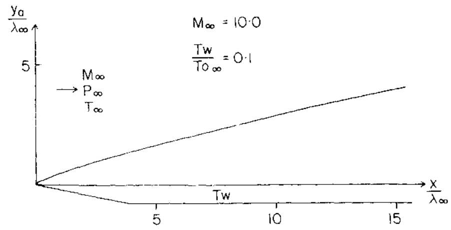

Fig. 3 The boundary of the shock-viscous layer region.

Furthermore, the pressure was assumed to be constant across this region.

It also was assumed that the fluid behaves as a perfect gas throughout the disturbed flow region, and the viscosity is proportional to square root of the temperature of the fluid.

Application of conservation principles resulted in two equations with the freestream Mach number M_∞, the wall temperature ratio T_w/T_{0_∞}, the slip parameter $y_a/c_1\lambda_b$, the degree of compression η, and the wave angle σ as parameters. These two equations have to be solved through a step-by-step procedure. For a flow problem with given M_∞ and T_w/T_{0_∞}, and at each value of the slip parameter $y_a/c_1\lambda_b$, both η and σ have to be iterated upon until the conservation principles are satisfied.

Calculations on such a flow problem based on the method described previously would result in a steady increase in the pressure from the freestream level at the very tip to a pressure plateau. Shortly after this maximum pressure has occurred, the degree of compression has essentially reached 100%, indicating the near completion of the compression to the states given by Rankine-Hugoniot relations. Afterwards, calculation procedures should be changed into a scheme, which is obtained with additional modifications introduced into an earlier formulation to be reported in Ref. 4.

Such calculations have been carried out for a flow case of $M_\infty = 10$, and $T_w/T_{0a} = 0.1$. Figure 1 shows the pressure distribution along the plate which increases from the freestream value at the very tip to a maximum and gradually decreases toward the downstream. Figure 2 shows the variation of the slip velocity φ_b at the wall, the degree of compression η of the shock-wave-like region, which is defined as $\eta = (1 - \xi)/(1 - \xi_0)$ where $\xi = v_n/v_{n_\infty}$, and ξ_0 is the value of ξ corresponding to the state given by Rankine-Hugoniot relations. Also shown in Fig. 2 are the wave angle and values of ξ_0 and ξ. Figure 3 shows the boundary along which the compression region is joined with the boundary-layer region. Agreement with the experimental data from Ref. 5 on pressure distribution (see Fig. 1) along the flat plate is quite good. Detailed formulation of the analysis on this problem will be reported in a forthcoming communication.

Based on the calculations described previously, the maximum pressures on the plate for various combinations of M_∞ and T_w/T_{0_∞} have been obtained and are shown in Fig. 4.

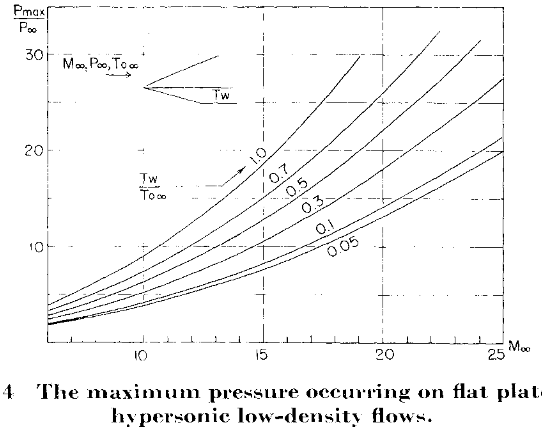

Fig. 4 The maximum pressure occurring on flat plates in hypersonic low-density flows.

A simple comparison shows that the plateau pressures reported by Oguchi[1] are higher than the values reported here at the same parametric values.

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Further Study on the Dynamic Stability of Cylindrical Shells

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Nomenclature

$\bar{a}_0$ = parameter of prebuckling deformation in the longitudinal direction
$\bar{a}_1$ = perturbation parameter in the longitudinal direction
$\bar{b}_1$ = perturbation parameter in the tangential direction
$\bar{c}_0$ = parameter of prebuckling deformation in radial direction
$\bar{c}_1$ = perturbation parameter in the radial direction
h = thickness of shell
l = length of shell
m = number of half-waves in the axial direction
n = number of half-waves in the circumferential direction
p_0 = external pressure
p_c = $E/[4(1 - \nu^2)](h/R)^2$ = critical external pressure (fo infinitely long shells)
R = radius of shell
τ = dimensionless time
τ_p = dimensionless time at the discontinuity of a ramp
u = displacement in x direction
α = see Eq. (33a)
γ = see Eq. (3a)
ν = Poisson's ratio
$'$ = $\partial/\partial\tau$

ONE important and interesting area of study on th dynamic stability of shells is the investigation of th influence of loading rate on the critical behavior of shell r sponses. This influence was illustrated in a recent paper b Bieniek, Fan, and Lackman (see Fig. 4 in Ref. 1). It ma be seen for shells of the same material, with given geometr and subject to identical magnitude of dynamically appli constant pressure that the higher loading rate tends to ma the thin shells more susceptible to instability. This co clusion was based upon the result of an approximate soluti of Eqs. (33) and (33a) from Ref. 1, which are repeated belo

$$\bar{c}_0 = -(p_0/p_c)[h/(\pi R)](1 - \alpha \cos\tau) \quad (3$$

where

$$\alpha = (\tau/\tau_p)[2 - 2\cos(\tau_p/\tau)]^{1/2} \quad (33$$

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